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DELAWARE ESTUARY COMPREHENSIVE STUDY. TECHNICAL REPORT NO. 5. ON THE MANAGEMENT OF THE QUALITY OF NATURAL WATER SYSTEMS

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Delaware Estuary Comprehensive Study

Report Number 5

Division of Water Supply and Pollution Control
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DELAWARE ESTUARY COMPREHENSIVE STUDY

TECHNICAL REPORT NO. 5

On the Management of the Quality of Natural Water Systems*

Introduction

In recent years publicity has been given to the mounting need to husband the nation's water resources as its population and industrial base expand [1]. The epithet "water-short", however, rarely means that an amount sufficient to satisfy an area's needs for a few days is unobtainable. Instead, it indicates a critical delay between the use of water and its subsequent availability in a state acceptable for reuse. Most domestic and industrial applications of water alter its quality by changing the temperature and adding to it solvents and solids. Given enough time, the original quality can be recovered through the natural hydrologic cycle. More immediately, many solids settle out, some organic wastes decompose biochemically, and some other pollutants react chemically.[1] Although municipalities and industries can hasten purification by artifically processing their waste fluids, the procedure requires time; and the longer it is possible to delay reuse, the greater is the minimum volume of water necessary to satisfy a given rate of demand.[2] For many purposes, therefore, the management of water resources corresponds to the control of the interval of time necessary for water quality to recover from the effects of pollutants. Thus, research on the management of water quality has pragmatic justification. Because any controls which are

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1. Detergents and phenols, for example, do not necessarily disappear for several months.
2. Even if all waste is reclaimed, "new" water must be added to make up for evaporation and other losses.

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applied interact with social values and economic needs, the technological alternatives ought to be subjected to an interdisciplinary examination. Operations Research seems well suited to this task.

The results of such research could exert enormous leverage in the direction of improved public welfare. This may be inferred from the scale of capital expenditures for the improvement, preservation, and control of water quality. Congress authorized the appropriation of up to one hundred million dollars in grants to municipalities for up to thirty percent of the construction costs for necessary sewage treatment facilities during the next fiscal year (PL 87-88, 84-660). This will be augmented by the construction of treatment plants under provisions of last year's Accelerated Public Works Act. State and local governments, public institutions, and industry also spend large sums for the treatment of their wastes.

The allocation of these investment funds within and among water resource systems and the design and operation of individual facilities are rich in operational problems, some of them familiar to Operations Research analysts. These include:

(i) The design of sewage treatment plants in the light of the variability of: inputs to them, stream flows and stream quality.

(ii) The programming of sewage treatment plant operations so that the effluents are good and the cost of the operations is minimal.

(iii) The allocation of funds among sites and facilities along a stream to attain a given improvement in stream quality at minimum cost.

(iv) The design of surveillance systems to warn of the advent of decreased quality.

(v) Use of utility theory to determine marginal social benefits as a function of water quality.

(vi) Optimal mix of investments in waste treatment and water treatment plants.

(vii) To what extent does investment in waste and water treatment facilities determine future regional development?

(viii) The optimal joint exploitation of ground and surface water resources.

The susceptibility of these problems to analysis is heightened by the wealth of published investigations of diffusion and decay by oceanographers, physicists, chemists, biochemists and others [4]. The literature is rich in experimental results and mathematically elegant theories. As has been shown [7], the mathematical elegance permits the application of control systems analysis; it will be demonstrated here that this leads to some answers to several of the operational problems above.

This paper describes several operational problems in water quality management whose analyses result from the manipulation of the differential equations describing the time rates of change in the concentrations of qualitative attributes. Because the equations describe a dynamic linear communication system, an introduction to the analysis of such systems precedes description of the water quality management problems. In order to give the reader an appreciation of the alternatives facing the manager of the water system, the problems will be presented in the context of

attempts to control a particular qualitative attribute; water will be considered as the vehicle for the oxygen dissolved in it. Dissolved oxygen (DO) has been studied extensively [3,7], its depletion often constitutes the primary problem in the quality of a water system and its control may be affected in many ways (rich in opportunities for analysis).

Quantitative and Qualitative Attributes

Dissolved oxygen (DO) is by no means the only noteworthy attribute of natural water systems. To begin with, one needs sufficient quantities of water. As noted in the Introduction to this paper, in many regions of the United States, the problem is one of having sufficient quantities at an acceptable quality rather than a sheer lack of water. This is nearly always the case for seaboard areas. Withdrawals of water from an estuary,[3] for example, would have to lower the ocean level before a quantity problem could exist. We shall ignore the quantity-quality interaction in this paper; assume that the natural water system of interest is an estuary.[4]

A time function associated with any point in an estuary is its history of concentrations of dissolved oxygen (in volume of oxygen per unit volume

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3. An estuary is that portion of a river which is influenced by the tides of the sea into which it empties. Some U. S. estuaries are the Delaware River from Trenton, N. J. to Delaware Bay and the Hudson River from Albany, N. Y. to New York City harbor.

4. Note that an estuary is a general case of which a straight-run stream is a particular example (tidal component equal to zero).

of water). This concentration can take any of a continuum of values from zero to a maximum governed by the water temperature, atmospheric pressure and water salinity.[5] For economic and aesthetic reasons, it is desirable to have a strictly positive concentration of DO everywhere in a natural water system. In particular, the nature of the biochemical process in which organic waste matter (dispersed in water) is decomposed is determined by whether or not oxygen is present in the region (of the water) within which decomposition occurs. Without oxygen, anaerobic bacteria thrive, and the decomposition is accompanied by hydrogen sulfide stenches and deposits of slime along the shores and on the surface of the water. Aesthetically, this outcome is objectionable because any natural scenic beauty has been destroyed.

The economic motivation to maintain a positive DO has its sources in the industries dependent upon recreational land use, commercial fisheries and the present and potential withdrawal uses of water. Some users of water who withdraw it from its natural courses are municipal water treatment plants, thermal electric generating stations (for cooling furnace walls), wood pulp processors and canneries. Water more economically satisfies their demands if its DO is not too low.

Other effects arise from the need of higher forms of aquatic life for oxygen. Relatively coarse fish, such as carp and catfish, need at least two to three milligrams per liter and game fish, such as trout, need higher concentrations; even plankton appear to require the presence of some oxygen to survive. Consequently, anaerobic decomposition of wastes also precludes

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5. This functional dependence assumes that plankton are absent. Photosynthetically produced oxygen can cause supersaturation which would not occur in streams whose DO is seriously depleted.

the use for recreational or commercial fishing.

Dynamic Linear Systems

Water quality control schemes now in use are founded upon static analyses. Formally, a decision is made as to what minimum concentration of DO is acceptable. Then the concentration that would result were there sources but no sinks of oxygen is computed. Finally, the sinks are controlled so that their net withdrawal of oxygen leaves the concentration at the minimum acceptable DO. The sinks whose control is intended are sewage effluents which would be dispersed in the water of the stream. A measure of the propensity of water to consume oxygen while being biochemically reduced is its biochemical oxygen demand, its "B.O.D." In the light of the net B.O.D. that would enter the stream were there no control, the fraction of B.O.D. is computed that must be removed by waste treatment processes so that the DO is no lower than the minimum allowable concentration. This fraction, a "percentage removal" of B.O.D., becomes a lower bound to the waste treatment before an effluent goes into the stream (when control is in effect).

The primary drawback to static analyses is caused by the time variation of effluents' B.O.D. and the induced time variation of the B.O.D. of material dispersed about any point in a stream. These varying stimuli cause variations in the DO, which responds to them. A static analysis, unfortunately, affords no opportunity to explain the time variations in DO as a function of its causes; it is not a causal analysis.

Fortunately, however, the diffusion process for scalar properties of water (one of which is DO) has been studied closely and its differential equations are known. These equations can be applied to a network of finite-

dimensional segments with which one may approximate any natural water system. Under several reasonable assumptions,[6] the equations are then linear, ordinary and of the first order. Consequently, one may, relatively easily, apply techniques developed for the analysis of dynamic linear communications system.[7]

The following aspects of dynamic linear systems are particularly relevant:[8]

(i) Oscillations produced in a linear dynamic system by a harmonic signal (input) are also harmonic functions of time, having the same angular frequency as the signal but differing from it in amplitude and phase; these differences, analytic functions of the frequency, are embodied in complex-valued "transfer functions." The transfer functions can be computed from the matrix of coefficients of the differential equations.

(ii) The transient responses to an input signal consisting of an impulse (Dirac Function) can be computed from the real parts of the transfer functions. These outputs are called "impulsive response functions."

(iii) The stability of the system can be analyzed from the solution to the homogeneous equations.

(i) and (ii) each afford a method of predicting the outputs for a set of inputs. One can, for instance, Fourier analyze each of the inputs and,

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6. The assumptions are:
   (a) The network of finite-dimensional segments, each segment homogeneous, approximates the natural water system.
   (b) Oxygen is transported into (or out of) a segment either by advection or diffusion.
   (c) Diffusion at a segment's border occurs at a rate proportional to the difference between the concentrations in the neighboring segments (i.e. a "gradient-exchange" model).
7. See Thomann [7] for the adaptation of linear systems analysis to an estuary and for a sophisticated analysis of DO time series.
8. See Chapter 1 of [6] for a concise presentation of the fundamentals of linear systems analysis.

because the system is linear, superimpose the outputs obtained by operating on the inputs with the transfer functions. Alternatively, each input may be assumed to be a sequence of impulses and, after integrating with the appropriate impulsive response function (the inverse Laplace transform of the transfer function), the results superimposed. Furthermore, the transfer functions (and, therefore, the impulsive response functions) depend only upon the matrix of coefficients (of the differential equations) not upon the inputs; i. e. they are inherent to the system.

Natural Water Systems as Dynamic Linear Systems

Assume that a natural water system consists of a network of finite-dimensional segments. When a signal, or input, is applied to any one of these, there is a response to it (DO) in each of the n segments; the functional relationship is embodied in a linear, ordinary, first-order differential equation in the n responses.

For the general case of an input at each segment, the total system is described by n equations in n responses. The coefficients are inherent to the system (i.e. independent of the inputs): they depend on the flow entering each segment, the volume of each segment, the turbulence within each segment (a constant governing the rate of diffusive transfer of oxygen across the border between two segments) and the reaeration rate (a constant that governs the rate of diffusive transfer of oxygen from the atmosphere into a segment).

Each input (or "forcing function") to the system is the sum of three functions of time: the amount of B.O.D. (the oxygen sink) dispersed in the segment, the net rate of photosynthetic production of oxygen in the segment (sometimes a sink) and the saturation concentration of D.O.

When a natural water system is assumed to be one-dimensional, the computation of the transfer functions is simpler than in the general case. The matrix of coefficients becomes tri-diagonal, facilitating the evaluation of determinants.

Points of Control

Control of DO can be exercised by varying the flows, volumes, reaeration and turbulence characteristics, rates of photosynthetic production, saturation concentrations and amounts of organic material (B.O.D.) dispersed throughout the segments of the estuary. The economics of the control of only the last three of these sets will be considered in this paper; the programming problem presented later is most easily set up with only the system's inputs as variables. In practice, furthermore, they often offer the most accessible avenues to the application of control.

Weighting (Impulsive Response) Functions and Supplemental Outputs

A measure of the delay between a signal (output) and its response is embodied in the transfer functions which can be written

$$\phi(j\omega) = A(\omega)e^{j\theta(\omega)}.$$

$\theta(\omega)$ is the phase shift caused by passing the signal through the system and $A(\omega)$ is the attentuation of the output amplitude relative to the input amplitude.

A transfer function, $\phi(j\omega)$, and corresponding impulsive response function, $k(t)$, can be represented by the transform pair:

$$k(t) = \int_{-\infty}^{\infty} \phi(j\omega)e^{j\omega t}d\omega, \qquad t > 0 \qquad (1)$$

$$\phi(j\omega) = \int_{0}^{\infty} e^{-j\omega t}k(t)dt \qquad (2)$$

where t is time, ω is frequency and $j = \sqrt{-1}$. Because $k(t)$ can be regarded as the response to a Dirac-delta input $\delta(t)$, and an arbitrary input, $f(t)$,

can be approximated by a sequence of pulses, each of the form $f(t_i)\Delta t_i \delta(t-t_i)$, the transient output, x(t), caused by f(t), is approximately

$$x(t) = \sum_i f(t_i)k(t-t_i)\Delta t_i. \tag{3}$$

By taking the limit as $\Delta t_i \to 0$,

$$x(t) = \int_{-\infty}^{\infty} k(t-\tau)f(\tau)d\tau \tag{4}$$

which can be restricted to

$$x(t) = \int_{-\infty}^{t} k(t-\tau)f(\tau)d\tau \tag{5}$$

because

$$k(t-\tau) = 0, \quad \tau > t. \tag{6}$$

(6) is the consequence of "physical realizability" of the system; i.e. a response cannot precede its cause.

It can be seen from (5) that the output, x(t), depends linearly on the values of the input, f(t), at all previous times, τ, with a weight $k(t-\tau)$ depending only on the time difference $t-\tau$; for this reason, k(t) is called a "weighting function." x(t), then, is a moving weighted average of the past inputs multiplied by a constant; the value of the constant is w[12] where

$$w = \int_0^{\infty} k(t)dt. \tag{7}$$

The response to a constant input f(t) = f, therefore, is

$$x = wf. \tag{8}$$

In practice, one need not know $f(t-\tau)$ for τ greater than some finite number, because if the integrals in (5) and (7) converge, for any $\epsilon > 0$,

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12. w can also be computed as w = A(0); $A(\omega)$ was defined above. Take limit as $\omega \to 0$ of

$$A(\omega)e^{j\theta(\omega)} = \phi(j\omega) = \int_0^{\infty} k(t)e^{-j\omega t}dt:$$

however small, there exists a finite R, $R > 0$, such that

$$\left| \int_{t-R}^{t} k(t-\tau)f(\tau)d\tau - \int_{-\infty}^{t} k(t-\tau)f(\tau)d\tau \right| < \epsilon \qquad (9)$$

and

$$\left| \int_{0}^{R} k(t)dt - \int_{0}^{\infty} k(t)dt \right| < \epsilon. \qquad (10)$$

Clearly, $f(\tau)$ is of no practical interest for $\tau < t-R$, R sufficient for a small ϵ in (9); i.e.

$$x(t) \cong \int_{t-R}^{t} k(t-\tau)f(\tau)d\tau. \qquad (11)$$

The preceding argument may be used to describe an operational problem that confronts the managers of many natural water systems. Figure 1 exhibits the time function of DO in a segment with whose quality there would be dissatisfaction. DO is at or near zero during the summer and fall and it would be preferable instead, to have the output (DO) not go below, say 4 mg/l; in Figure 1, the dashed line represents this minimum level. The "supplemental output" which would have to have been superimposed upon the existing output so that the sum of the two would have been, at no time, less than 4 mg/l, is indicated in the lower part of Figure 1. Assume that the signals which acted upon the system to produce this recorded response and the transfer functions through which they operated are all known (through (1), the

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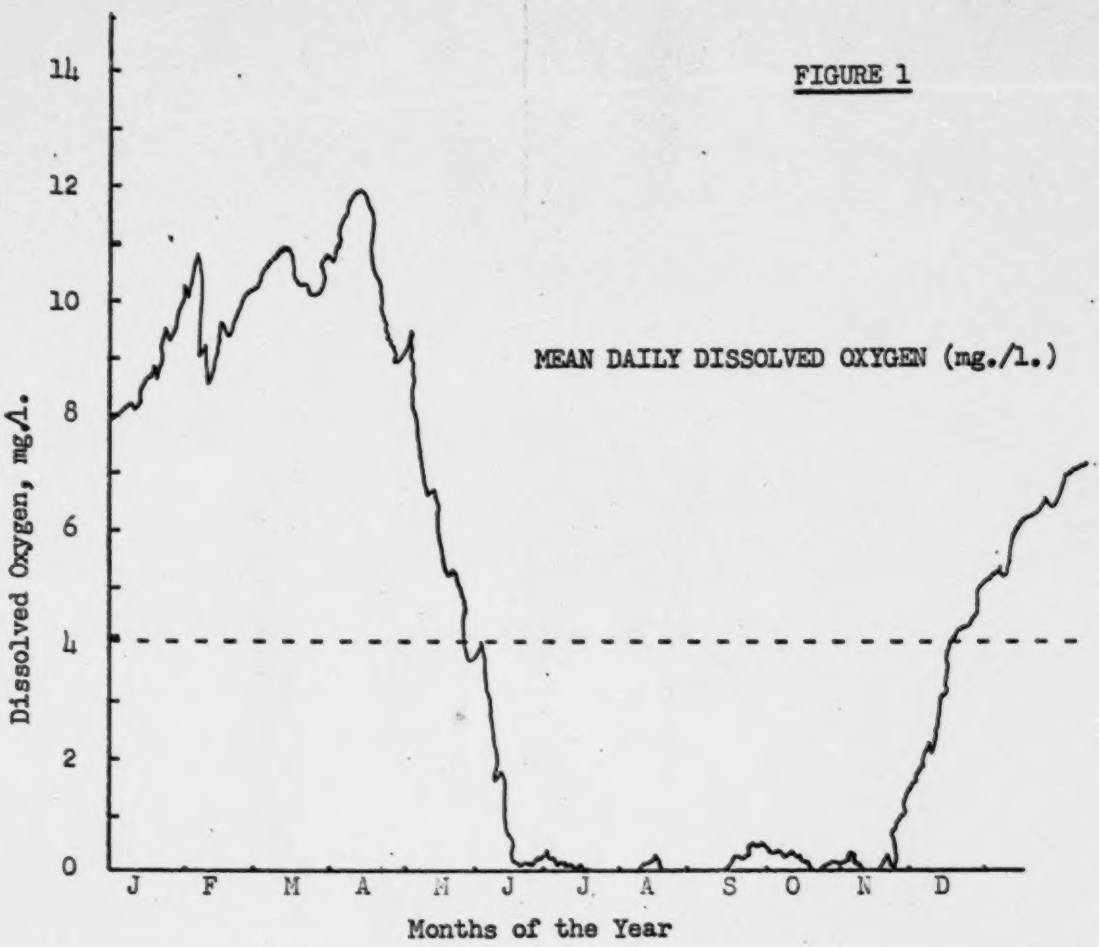
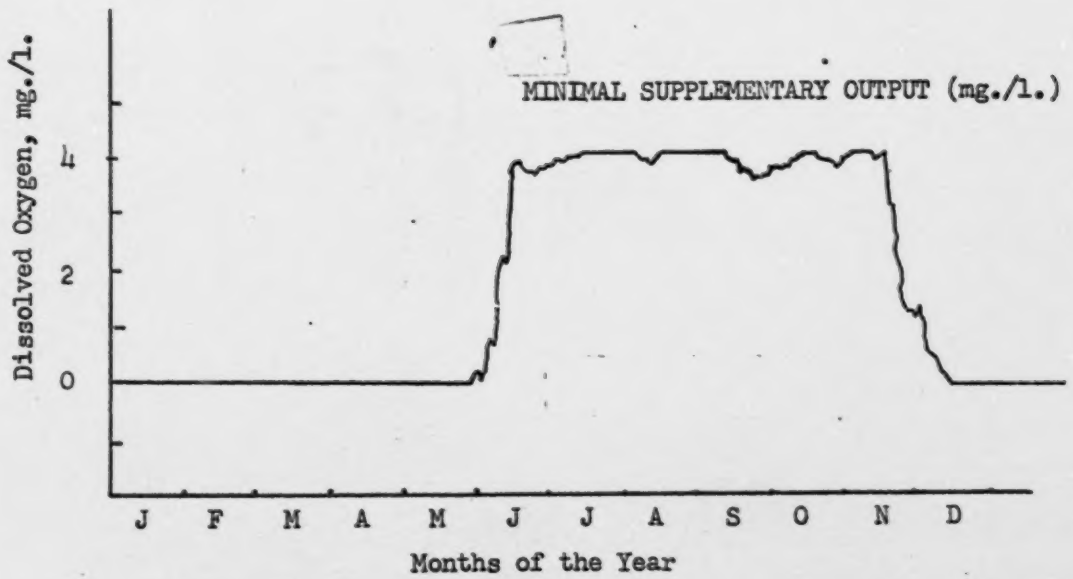
12 (continued).

$$A(0)e^{j\theta(0)} = \int_{0}^{\infty} k(t)dt = w$$

and $A(0) = w$ because $\theta(0) = 0$ by its definition on p. 5 of [6].

FIGURE 1


MEAN DAILY DISSOLVED OXYGEN (mg./l.)
Dissolved Oxygen, mg./l.
14
12
10
8
6
4
2
0
J F M A M J J A S O N D
Months of the Year


MINIMAL SUPPLEMENTARY OUTPUT (mg./l.)
Dissolved Oxygen, mg./l.
4
2
0
J F M A M J J A S O N D
Months of the Year

weighting functions are then also known). Let:

$\{\tilde{f}_i(t)\}$ = the signals which are applied at the i-th segment, i = 1,...,n;

$\{k_{iv}(t-z)\}$ = the set of impulsive response (weighting) functions for a signal applied to the i-th segment at time z and responded to in the v-th segment at time t, i = 1,...,n, v = 1,..,n and t > z;

$\{\tilde{c}_v(t)\}$ = the set of DO time functions which are the responses to $\{\tilde{f}_i(t)\}$, i = 1,...,n.

Then

$$\tilde{c}_v(t) = \sum_{i=1}^{n} \int_0^t f_i(z) k_{iv}(t-z)dz, \qquad v = 1,...,n. \tag{12}$$

(12) follows from the fact that the response in the v-th section to a signal in the i-th section is

$$\overset{i}{c}_v(t) = \int_0^t f_i(z) k_{iv}(t-z)dz \tag{13}$$

and

$$c_v(t) = \sum_{i=1}^{n} \overset{i}{c}_v(t), \qquad i = 1,..,n;\ v = 1,..,n. \tag{14}$$

The problem is: what ought the signals have been to produce a DO record in v_1 that we would have liked to see? Because the system is linear, one need consider only the supplementary signals which, when added to $\{\tilde{f}_i(t)\}$, would produce a minimally acceptable output. Let:

$C_{v_1}(t)$ = the minimally acceptable output in segment v_1;

$c^*_{v_1}(t) = C_{v_1}(t) - \tilde{c}_{v_1}(t)$ = the minimally acceptable supplementary output;

$\{f_i(t)\}$ = any set of signals whose net response in segment v_1 has a lower bound of $c^*_{v_1}(t)$;

$\{c_v(t)\}$ = the set of responses to $\{f_i(t)\}$.

Then, in general,

$$c_v(t) = \sum_{i=1}^{n} \int_0^t f_i(z) k_{iv}(t-z)dz, \quad v = 1,\ldots,n \tag{15}$$

From (12), (15) and the definition of $c^*_{v_1}(t)$,

$$\begin{aligned} C_{v_1}(t) &= \tilde{c}_{v_1}(t) + c^*_{v_1}(t) \\ &\leq \sum_{i=1}^{n} \int_0^t \left[f_v(t) + f_v(t)\right] k_{iv}(t-z)dz. \end{aligned} \tag{16}$$

The problem to be analyzed is the determination of $\{f_i(t)\}$ for the kind of $c^*_{v_1}(t)$ seen in Figure 3, when certain cost functions are applicable.

The nature of the problem and its analysis depend, in part, upon the $c^*_{v_1}(t)$ whose attainment is sought. For example, the alternative counteractions that may be taken against the transient effects of a city's storm sewers already overflowing into an estuary because of local

cloudbursts is much more limited than the ways in which the perennial sag in DO during the summer and autumn (seen in Figure 1) may be boosted. These will be designated as transient or short-term control and seasonal or long-term control problems; the remainder of this paper will dwell on seasonal control. Should the recorded output shown in Figure 1 be taken as that of section v_1 and 4 mg/l be the minimally acceptable output ($C_{v_1}(t)$), it is clear that $c^*_{v_1}(t)$ is approximately a trapezoid which extends from the beginning of summer through autumn. Figure 1 is representative of the DO response in segments of some natural water systems whose qualities need improvement and the trapezoid five or six months in length is representative of the type of supplemental output that needs to be generated in various segments of these systems. In any particular system, a set $\{f_v(t)\}$ which would generate a trapezoid is not unique (non-uniqueness would be true, generally, for any $c^*_{v_1}(t)$). In fact, first it must be verified that there exists at least one feasible set $\{f_i(t)\}$ (capable of physical attainment). Generally, the nature of various sets, $\{f_i(t)\}$, whose net response in v_1 would have a lower bound consisting of a specified trapezoid depends upon the characteristics of $\{k_{iv_1}(t-z)\}$.

The Steady State: A Programming Problem

It is useful to know how to allocate investment funds so that a desired improvement in water quality will be attained at minimum cost. In the notation of the preceding section, a set $\{f_i(t)\}$ is sought which will attain $c^*_{v_1}(t)$ at the minimum cost of so doing. It will

be assumed in this section that $c^*_{v_1}(t)$ and $\{f_i(t)\}$ are constants and their role in a stable system in the steady state is being analyzed.

Let:

$m_i = f_i(t)$, $i = 1,\ldots,n$, for all t;

$b_{v_1} = c^*_{v_1}(t)$, for all t;

$$w_{iv} = \int_0^\infty k_{iv}(t)dt, \qquad i = 1,\ldots,n;\ v = 1,\ldots,n.$$

In the steady state, $m_i w_{iv}$ is the response in segment v to $f_i(t)$. The problem is to find $\{m_i\}$ such that $\sum_{i=1}^{n} m_i w_{iv_1} \geq b_{v_1}$, at the minimum cost of so doing.

m_i, i=1,..,n, will be constrained to be non-negative; i.e. a decrease in quality in some segments will not be accepted merely to reduce the cost of generating an improvement in segment v_1.

The cost of generating an output is the cost associated with $\{m_i\}$. It will be assumed that this cost is $\sum_{i=1}^{n} V_i(m_i)$. In other words, a particular cost function, $V_i(m_i)$, applies to each segment, but it is independent of the values of $m_1,\ldots,m_{i-1},m_{i+1},\ldots,m_n$. As posed so far, the problem is to find $m_1,\ldots,m_n$ to

$$\text{minimize} \quad \sum_{i=1}^{n} V_i(m_i)$$

$$\text{subject to} \quad \sum_{i=1}^{n} m_i w_{iv_l} \geq b_{v_l} \qquad (17)$$

$$\text{and} \quad m_i \geq 0, \quad i = 1,\ldots,n.$$

Several considerations ruin the potential simplicity of this formulation. The science of biology has not yet advanced to the stage where an exhaustive causal analysis can be made of photosynthetic production of oxygen. Therefore, the control alternatives are reduced in number. The labor necessary to solve (17) is heightened by the nonlinearity of $V_i(m_i)$, $i = 1,\ldots,n$ (and by the fact that there does not exist, generally, a single transformation that would linearize all of $\{V_i(m_i)\}$). Finally, parameters of the transfer functions (flow, turbulence and reaeration coefficients) are not variables to be jointly optimized with the inputs.[9]

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9. Parameters may be included if empirical transfer functions can be computed for them: $\emptyset(j\omega)$ can also be represented as $X(j\omega)/F(j\omega)$, the ratio of the Fourier transforms of the output and input, respectively. Therefore, a portion of $x(t)$ must be sought for which there was significant variation in the parameter of interest but non in the inputs and other parameters; those portions of the parameter's time function and the output would be Fourier analyzed and the ratio $X(j\omega)/F_{\text{parameter}}(j\omega)$ computed.

   This convenient computation of a parameter's transfer function assumes that the output responds linearly to the parameter, an assumption that is often untenable.

There may be several segments of the estuary with whose DO time functions there is dissatisfaction. Say that there are M such segments; then (17) may be generalized:

$$\text{find } \{m_i\} \qquad \text{to minimize} \qquad \sum_{i=1}^{n} V_i(m_i)$$

$$\text{subject to} \quad \begin{pmatrix} w_{1v_1} & \cdots & w_{nv_1} \\ \vdots & & \vdots \\ w_{1v_M} & \cdots & w_{nv_M} \end{pmatrix} \begin{pmatrix} m_1 \\ \vdots \\ m_n \end{pmatrix} \geq \begin{pmatrix} b_{v_1} \\ \vdots \\ b_{v_M} \end{pmatrix} \tag{18}$$

and $m_i \geq 0$, $i = 1,..,n$.

Within its limitations (18) is a formalization of the problem of attaining specified improvements in water quality at the least possible cost. Use was made of the system's dynamic characteristics in order to pose the (static) problem of increasing the mean quality at the minimum cost. Increases in the mean quality, nevertheless, are the ends to which virtually all investments in facilities (primarily waste treatment plants) have been directed.

The greatest obstacle to the use of (18) as a starting point for "optimal" investment computations is present ignorance of the marginal cost functions. Only occasional analysis has been directed at their specification [2].

Heretofore, water quality has not been a constraint in the formulation of problems of optimal multi-purpose development of water resources. With the constraints linear as in (18), however, this may be possible in the future.

Conclusions

1. Social and economic importance will be attached to solutions of operational problems in water quality management.
2. Numerous problems exist which are similar to those arising in more familiar domains of Operations Research activity.
3. There exist tractable mathematical forms into which water quality systems may be put.
4. Solution of the steady state investment problem (least cost allocation of investments to attain given increases in the mean quality) awaits specification of the marginal cost functions of alternative investments.

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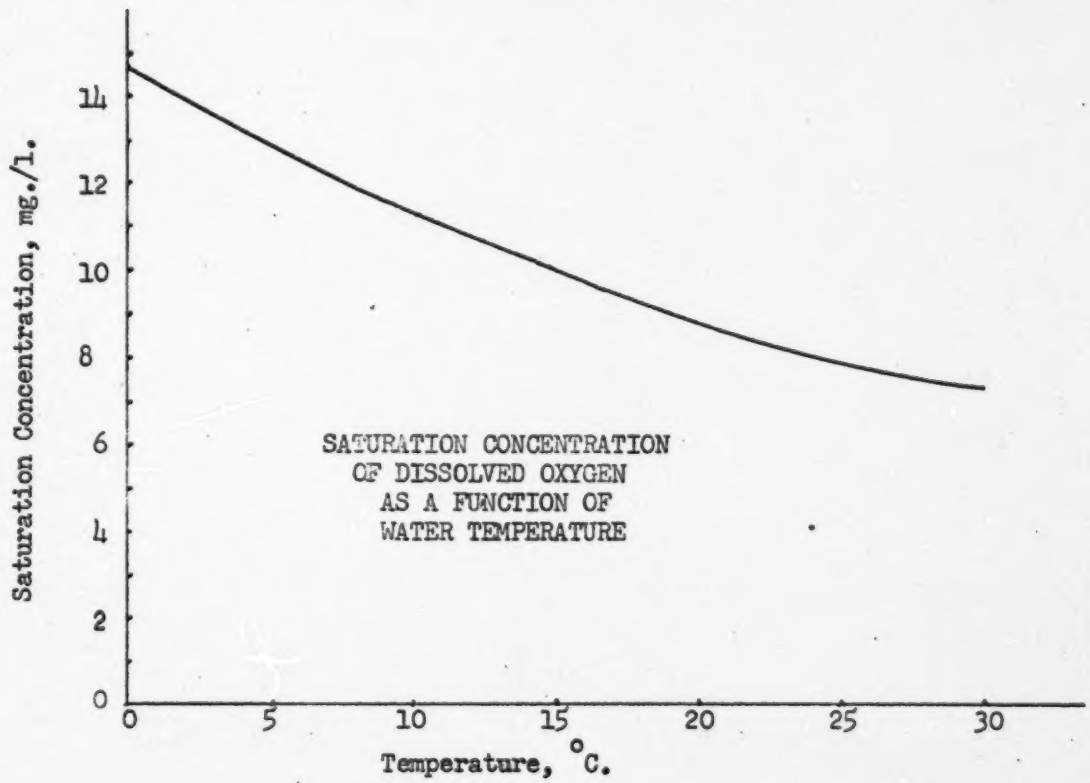
Appendix I: Sub-optimization

So that sub-optimizati[illegible] be avoided, it is logical to ask if control measures taken to [illegible] DO will adversely affect other qualitative characteristics of an estuary. Of these others, it is possible to affect only salinity, temperature and turbidity.

Turbidity, an optical property, is caused by the presence of suspended matter in water. Increasing the fresh water flow entering an estuary, an action which may increase DO in places, will stir up the bottom to some extent and increase turbidity. The time of the year, however, when one considers augmenting the flow is during the summer, when both the flow and the DO concentrations are at their lowest. Even the doubling of such low flows will not significantly increase turbidity.

The saturation concentration of DO is a convex decreasing function of water temperature (Figure A-1). By lowering water temperature, therefore, DO will be increased (because the rate of absorption from the atmosphere is proportional to the difference between the saturation and existing concentrations). It is not likely, nevertheless, that temperature control will be applied. Because the saturation concentration function is convex decreasing, at the high temperatures common during the summer, one would need temperature decreases in the order of 5°C to increase the saturation concentration by 1 mg/l. Even should this be a feasible engineering feat, biological considerations dictate against a point "source" of heat withdrawal such as an iceberg. It would constitute a severe shock to the aquatic life passing near it, thus causing extensive fish kills. We have not yet explored the economics of constructing and operating heat exchanging devices at a sequence (or continuum) of points.

FIGURE A-1



SATURATION CONCENTRATION
OF DISSOLVED OXYGEN
AS A FUNCTION OF
WATER TEMPERATURE

It has been mentioned (in a previous paragraph) that increasing the fresh water flow may increase the DO at some points. The flow also affects the salinity at various points; it acts as a hydrodynamic barrier against salt water intrusion which may be moved downstream by increasing the flow. Certain kinds of aquatic life, such as oysters, are able to survive in only a narrow range of salinity and may not thrive if fresh water flow increases, thus lowering the salinity everywhere in the estuary. The loss this represents, however, may be more than offset by the reduced costs of treatment for drinking water and industrial process water whose salinity must be made negligible. Such an economic analysis has not yet been completed. Flow regulation, furthermore, would sharply reduce the between year variation of summer low flows (and, therefore, variation of salinity) so that oyster farming might, in the long run when re-established, show a net benefit.

Thus, it seems warranted to consider the control of dissolved oxygen concentrations apart from the control of other qualitative characteristics.